# Short Papers

## Observations on Resonant Cavity Perturbation by Dielectric Objects

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*Abstract*—**Microwave resonators can be used as sensors for sorting dielectric objects. Differences in volume or mass can be distinguished among objects of irregular shape but of uniform dielectric properties. Moisture content, or other permittivity-dependent qualities can also be distinguished independent of mass or volume when objects are of uniform shape. Fundamental principles of the shape-independent and size-independent measurements are discussed, and experimental results are presented.**

### I. Introduction

Sorting dielectric objects of irregular shape according to volume, detecting internal voids in objects of similar shape, or determining moisture content in objects of varying volumes are important in the production, processing or storage of such objects. There are many nondestructive techniques available to accomplish these tasks with optical sensors, balances, mechanical sensors, and X-ray exposures, but they lack the required accuracy, have limited speed of operation, or are too susceptible to environmental hazards.

Resonant cavity techniques are widely used for determining microwave properties of materials [1] by measuring the shift of resonant frequency and the change in the $Q$-factor of the cavity when the sample is inserted into the cavity. Parameters of the cavity depend upon the volume, geometry, and mode of operation of the cavity, as well as on the permittivity, shape, dimensions, and location of the object inside the cavity. For a given cavity and material sample of regular shape and well defined dimensions, one can determine the permittivity of the material [2]. In this paper, we consider the possibility of sorting objects with shapes and dimensions that are unknown at the time of measurement.

Microwave resonant cavities have been used for evaluating the dielectric properties of geometrically defined samples [2], [3] when the cavity is calibrated with dimensionally identical samples of various known permittivities. By measuring a fiber in two resonant cavities, the dielectric constant and diameter of the fiber may be determined [4], or its moisture content may be determined independent of its diameter or density [5]. Finally, a single resonant cavity has been applied for determining moisture content in uniformly shaped seeds by simultaneous measurement of resonant frequency shift and change in the transmission factor [6]. The promising results obtained in that work provided a basis for continuing the research with other objects.

### II. Theoretical Considerations

When a small dielectric object (nonmagnetic with $\mu = 1 - j0$) of volume $v_s$ is placed in a resonant cavity having volume $v_c$ which has the electric field $\boldsymbol{E}_0$ and the magnetic field $\boldsymbol{H}_0$ in its unperturbed mode, the fields in the interior of the object are $\boldsymbol{E}$ and $\boldsymbol{H}$. The change in the resonant angular frequency may be expressed as [7], [8]

$$\frac{\omega_s - \omega_0}{\omega_0} = -(\epsilon - 1)\frac{\int \boldsymbol{E}\boldsymbol{E}_0^* \, dv}{2\int |E_0|^2 \, dv}, \tag{1}$$

where $\epsilon$ is the complex relative permittivity. Two approximations are made in applying (1), based on assumptions that the fields in the empty part of the cavity are negligibly changed by insertion of the object, and that the fields in the object are uniform over its volume. Both of these assumptions can be considered valid if the object is sufficiently small relative to the resonant wavelength. The negative sign in (1) indicates that by introducing the dielectric object the resonant frequency is lowered.

Because the permittivity of practical materials is complex; i.e., $\epsilon = \epsilon' - j\epsilon''$, where $\epsilon'$ is the dielectric constant and $\epsilon''$ is the loss factor of the material, the resonant angular frequency should also be considered as complex. Let $Q_0$ be the $Q$-factor of the cavity in the unperturbed condition and $Q_s$ be the the $Q$-factor of the cavity loaded with the object. Then the complex resonant angular frequency, $\Omega_0$ may be defined [8] as

$$\Omega_0 = \omega_0(1 + j/2Q_0). \tag{2}$$

With the object located in the cavity, the complex resonant frequency changes from $\Omega_0$ to $\Omega_0 + \delta\Omega$, with real part being $\omega_0 + \delta\omega$. Then

$$\Omega_0 + \delta\Omega = (\omega_0 + \delta\omega)(1 + j/2Q_s). \tag{3}$$

Subtracting (2) from (3), and assuming that changes in resonant frequency and $Q$-factor are relatively small, one obtains

$$\frac{\delta\Omega}{\omega_0} = \frac{\omega_s - \omega_0}{\omega_0} + j\frac{1}{2}\left(\frac{1}{Q_s} - \frac{1}{Q_0}\right) = \frac{f_s - f_0}{f_0} + j\frac{1}{2Q_0}\left(\frac{Q_0}{Q_s} - 1\right)$$
$$= -\frac{f_0 - f_s}{f_0} + j\frac{\Delta T}{2Q_0}. \tag{4}$$

Introducing the shift of the real component of the resonant frequency $\Delta F = f_0 - f_s$ and the transmission factor $\Delta T = 10^k - 1$, where $k = (S_{210} - S_{21s})/20$, $S_{21}$ is the voltage transmission coefficient at resonance, expressed in decibels, and subscripts 0 and $s$ refer to the empty cavity and the cavity loaded with the object, and comparing the resonant frequency shifts described by (1) and (4), one obtains

$$-\frac{\Delta F}{f_0} + j\frac{\Delta T}{2Q_0} = -\frac{1}{2}\left(\frac{1}{C}\right)K(\epsilon - 1)\left(\frac{v_s}{v_c}\right). \tag{5}$$

The value of the shape factor $K$ is dependent upon object shape, orientation and permittivity, but it can be determined for some regular shapes [9], and such values are listed in Table I. The parameter $C$ can be determined by

$$C = \frac{1}{v_c}\int \frac{|E_0|^2}{|E_{0\max}|^2} \, dv.$$

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TABLE I
VALUES OF THE SHAPE FACTOR $K$ FOR OBJECTS OF SIMPLE SHAPE

| Shape and Configuration | $K$ |
|---|---|
| Rod, bar $\parallel E_0 (h = b)$* | 1 |
| Strip, disk | $1/\epsilon$ |
| Sphere | $3/(\epsilon + 2)$ |

*$h$ is the height of the bar and $b$ is the height of rectangular waveguide.

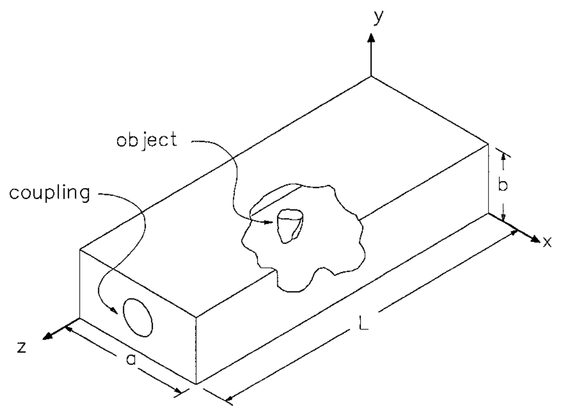


Fig. 1. Rectangular waveguide resonant cavity with an arbitrarily shaped dielectric object at the center.

For a rectangular cavity (see Fig. 1) operating in the $TE_{10p}$ ($H_{10p}$) mode, $E_0$ is given by

$$E_0 = E_{0\max} \sin \frac{\pi x}{a} \sin \frac{p\pi z}{L},$$

where $a$ and $L$ are the width and length of the cavity, and $C = 1/4$.

Equating real and imaginary parts in (5), one finally obtains the following expressions for the shift of resonant frequency and change in the transmission factor of the resonant cavity when loaded with an object of arbitrary shape [2], [9]:

$$\Delta F = 2(\epsilon' - 1) K f_0 \left(\frac{v_s}{v_c}\right) \tag{6}$$

$$\Delta T = 4\epsilon'' K^2 Q_0 \left(\frac{v_s}{v_c}\right). \tag{7}$$

In general, these relationships have been used for material permittivity measurements, provided the object is well defined in dimensions and shape; i.e., the values of $f_0$, $Q_0$ and $v_c$ are known for a given cavity, and the values of $K$ and $v_s$ are precisely determined for a sample of material.

Examination of (6) and (7) reveals, however, that they can be used together for evaluation of the properties of the object even when the volume of the object ($v_s$) is unknown prior to measurement. The ratio of those two parameters is

$$X = \frac{\Delta F}{\Delta T} = \frac{\epsilon' - 1}{\epsilon''} \frac{1}{K} \frac{f_0}{2Q_0} \tag{8}$$

which does not depend upon the volume of the object, but it is dependent on the shape of the object and its dielectric properties. The variable $X$ is density-independent or size-independent for objects of similar shape, and can be determined as a function of the permittivity-dependent properties of the object (density, moisture content, etc.). In practice, slight changes in shape will contribute to measurement errors [6].

TABLE II
PARAMETERS OF RECTANGULAR WAVEGUIDE RESONANT CAVITIES USED IN THIS STUDY

| Parameter | $S$-band $H_{105}$ | $C$-band $H_{107}$ |
|---|---|---|
| Inside dimensions | 72.14 × 34.04 mm | 47.55 × 22.15 mm |
| Length, $L$ | 305 mm | 203 mm |
| Resonant frequency | 3205.7 MHz | 6017.4 MHz |
| $Q$-factor, $Q_0$ | 1650 | 830 |
| Coupling aperture diameter | 20.0 mm | 13.0 mm |
| Volume | 749 cm$^3$ | 214 cm$^3$ |

Another possibility of applying (6) and (7) is a shape-independent ratio

$$R = \frac{\Delta F}{\sqrt{\Delta T}} = \frac{\epsilon' - 1}{\sqrt{\epsilon''}} \frac{f_0}{\sqrt{v_c Q_0}} \sqrt{v_s} \tag{9}$$

which can be rearranged into the following expression

$$m = v_s \rho = \left(\frac{R}{A}\right)^2 \tag{10}$$

where $m$ is the mass of the object, $\rho$ is its density, and $A$ is a constant depending on the cavity and on the permittivity of the material, which relates the measured quantities to the volume of the object $v_s$. (10) can be used for the volume (or weight) determination of objects of arbitrary shape with arbitrary orientation in the cavity, as long as one can assume that the dielectric properties of the material remain unchanged. In the next section, experimental evidence is presented that validates these statements.

## III. EXPERIMENTAL RESULTS

Resonant cavities consisted of sections of standard rectangular waveguides, coupled with the external waveguides through small apertures in short-circuiting metal plates. Parameters of two cavities used in this project are listed in Table II. Each cavity was placed between two waveguide-to-coaxial transitions which allowed it to be connected to an automatic network analyzer calibrated in the transmission mode. we used 801 discrete frequencies within a range of 8 or 16 MHz to determine the resonant frequency within increments of 10 kHz or 20 kHz, respectively. A "marker to maximum" command accomplished the determination of the resonant frequency with an accuracy better than 5 or 10 kHz, and the voltage transmission coefficient ($S_{21}$) with an accuracy of 0.02 dB.

The objects were introduced into the cavity through circular holes milled in the wide wall at the center of the waveguide section. The holes had diameters smaller than the cutoff wavelengths of circular waveguides and were filled with tightly fittings caps machined to close them after the objects were inserted. Twenty-one regularly shaped elements (spheres, disks, cuboids, and rods of lengths smaller than the height of the waveguide) of polyvinylidene fluoride (PVDF), known as Kynar, were tested in the $S$-band cavity. The permittivity of this material at 3.2 GHz, determined from the resonator measurements, was $\epsilon = 2.78 - j0.17$, which agreed well with available data [10]. Density of PVDF is 1.767 gcm$^{-3}$. Results of these measurements are presented in Fig. 2. The straight line has a slope $A = 9.34$ that is very close to the value 9.365 calculated ((9) and (10)) from the cavity parameters listed in Table II and the

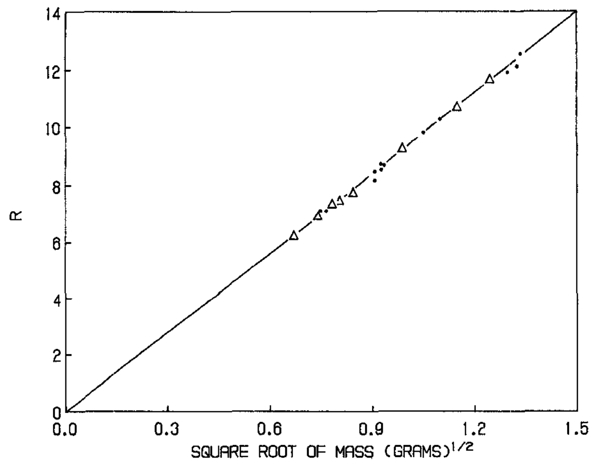


Fig. 2. Experimental results for polyvinylidene fluoride (PVDF) elements of various shapes in the $S$-band resonant cavity.

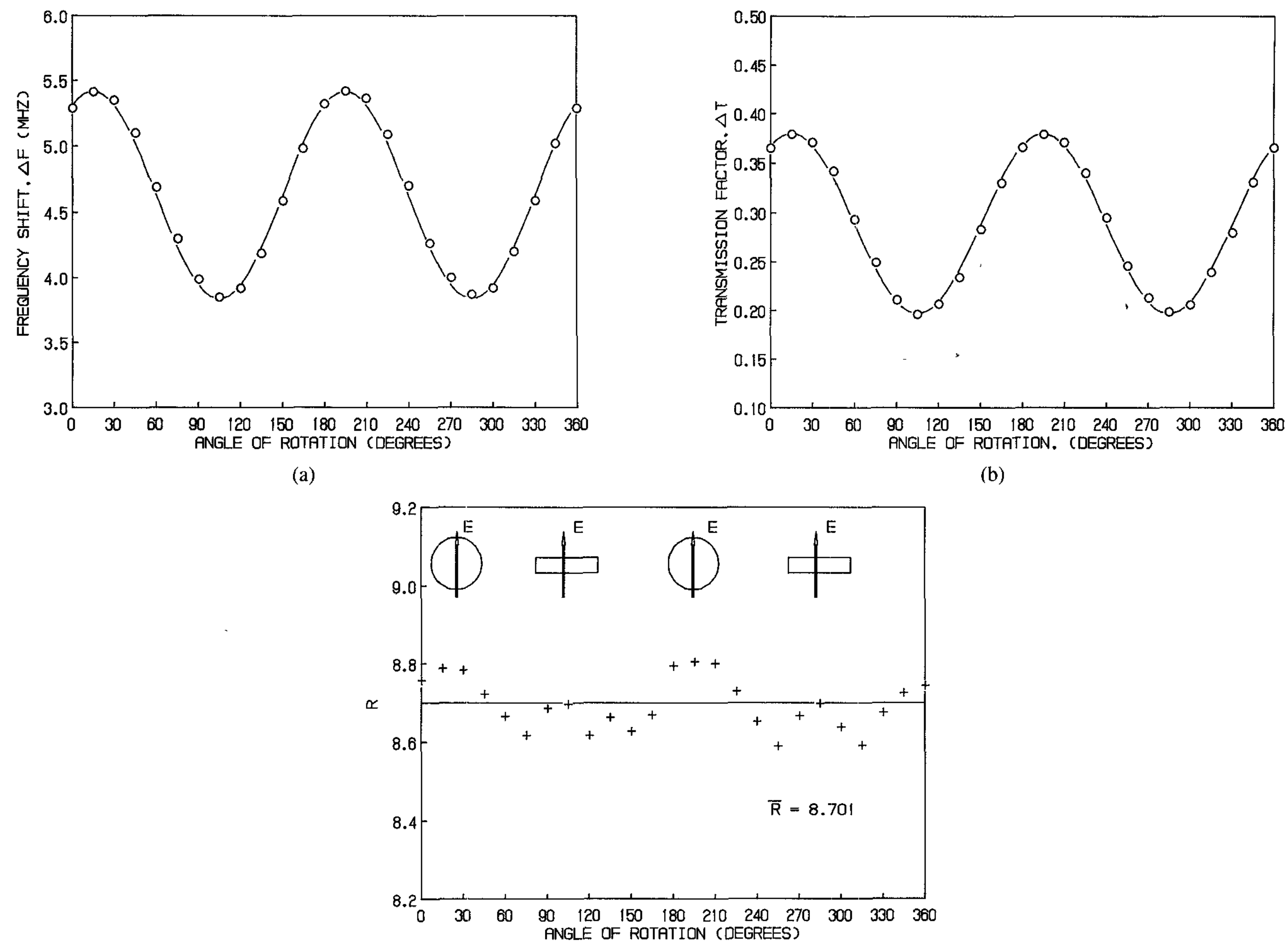


Fig. 3. Experimental results for PVDF disk of 11.76 mm diameter and thickness of 4.60 mm rotated about $x$-axis. Values of $\Delta F$, $\Delta T$, $R$, and disk positions are shown in (a), (b), and (c), respectively.

given material permittivity. What is even more interesting, the variable $R$ remains almost unchanged for a given sample regardless of sample orientation at the center of the cavity. Rotation of a disk-like sample with respect to the $x$-axis of the cavity, as shown in Fig. 3, provides significant changes in the $\Delta F$ and $\Delta T$, but values of $R$ remain almost constant within the uncertainty of measurement. Similar results have been obtained for rotation of the other samples of regular shape. However, for a sphere shifted along the x-axis, as shown in Fig. 4, values of $\Delta F$ and $\Delta T$ follow $\sin^2 \Theta$ behavior, and their ratio $X$ is constant within the measurement uncertainty.

Verification of the shape-independent properties of the variable

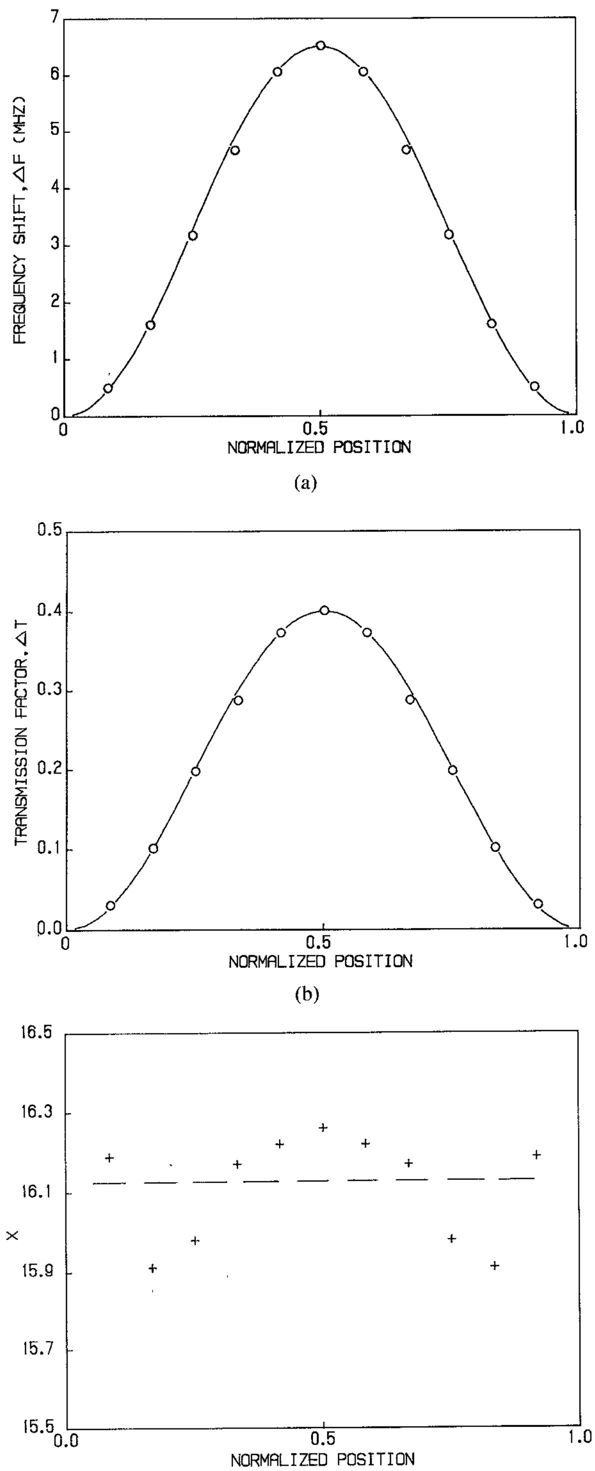


Fig. 4. Experimental results for PVDF sphere of 10.87 mm diameter shifted across the waveguide along $x$-axis. Values of, $\Delta F$, $\Delta T$, and $X$ are shown in (a), (b), and (c), respectively.

R was provided by the measurements of eight irregularly shaped samples of the same material and corresponding results are indicated by the triangular symbols in Fig. 2. The maximum difference between the measured mass and values calculated by (10) for the objects was 1.5%, whereas the average difference was 0.35% (approx. 0.003 g). Accuracy in determining the volume of the object is of the same magnitude.

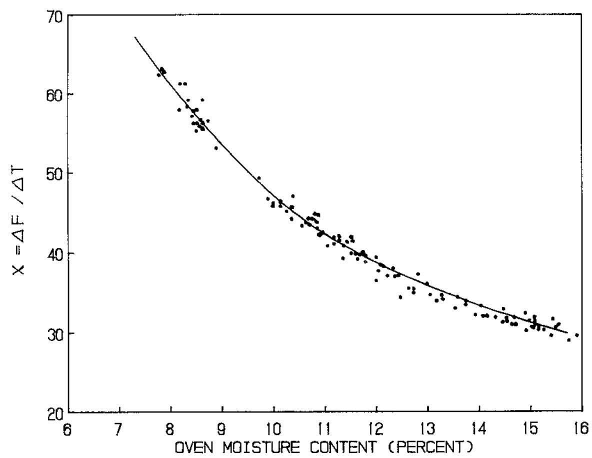


Fig. 5. Experimental results for soybean seeds for various sizes and moisture contents in $C$-band rectangular cavity.

Other measurements were taken with soybeans which are seeds of nearly spherical shape, and the ratio of the major and minor diameters of the lot selected ranged from 1.2 to 1.36 [6]. Major diameters ranged between 6.5 and 7.5 mm. Seeds of different moisture contents and various sizes were measured, and the coefficient $X$ was calculated for each case according to (8). The dependence of the parameter $X$ on seed moisture content M determined by a standard oven drying method, for the cavity operating in the $C$-band is shown in Fig. 5. A simple empirical expression which fits a set of 108 experimental data points has the form $1/X = 0.00233\text{M} - 0.0021$ with a correlation coefficient $r = 0.991$. Inspection of the expression implies the calibration equation for moisture content determination in the form

$$M = \frac{430}{X} + 0.9. \tag{11}$$

The validity of this calibration equation was tested with 55 other soybeans, and the mean value of the difference between oven moisture content and moisture content calculated from (11) was 0.15% moisture. The standard deviation of the difference (standard error of performance) was 0.51% moisture.

## IV. Discussion and Conclusion

Microwave resonant cavities are potentially useful tools for sorting dielectric objects. The objects should be stopped individually at the center of the cavity for a short time period (about 20 ms or less) to permit the measurement. With empty cavity as a reference, the shift of resonant frequency and change in the transmission factor of the cavity when loaded with the object are measured simultaneously. The ratio of these two cavity parameters may be used as a size-independent function for objects of similar shape or a shape-independent function for objects of similar material properties. For a particular class of objects, the output from the measurement may be easily calibrated against a standard method for determining volume, density or moisture content, providing a direct reading of the desired properties.

The resonant-cavity measurement procedure is simple, fast, nondestructive and potentially a continuous method for sorting dielectric objects according to their properties (volume, density, moisture content, etc.). The object should be relatively small compared with the wavelength, allowing perturbation theory to be applied for simplification of requirements and calculations. The method requires that the object be positioned in a region of maximum electric field and that the relative complex permeability of the material be of unit magnitude. Limitations for practical sorting systems in terms of accuracy, stability, and object properties are to be determined. Also, a variety of other types of microwave resonators could be used for this purpose.

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# Efficient Computation of High-Frequency Two-Dimensional Effects in Multiconductor Printed Interconnects

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*Abstract*—**The spectral domain technique with a Galerkin moment method solution is used to study high-frequency, two-dimensional effects such as dispersion and leakage in multiconductor printed interconnects. A simple asymptotic procedure is used to significantly improve the convergence of oscillatory spectral integrals involving distant expansion and testing functions. Examples are given for leaky waves on two multiconductor printed transmission line geometries.**

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## I. Introduction

Multiconductor transmission lines have been the interest of numerous researchers for several decades. Recently, there has been much interest in multiconductor printed transmission lines used as interconnects in high-speed integrated circuits [1], [2]. In this paper, concentration will be placed on developing an asymptotic technique for the efficient numerical analysis of high-frequency, two-dimensional effects such as dispersion and leakage in printed interconnects.

The numerical analysis is formulatd in the spectral domain with a Galerkin moment method solution [1], [2]. Although this method is well known, special care must be taken when studying high-frequency effects such as dispersion and leakage. The reaction integrals which must be evaluated in the moment method solution are often highly oscillatory. This is especially true for the problem at hand: since a multiconductor system is considered, expansion and testing functions will often be relatively far apart in space. This coupled with the fact that leakage and dispersion usually are important at relatively high frequencies, leads to spectral integrals which are often highly oscillatory and hence CPU intensive.

Oscillatory integrals occur often in electromagnetic problems and therefore several asymptotic techniques have been developed for their evaluation. Much previous work has been directed at the investigation of three-dimensional layered problems and has led to analyses which, although general, are more complicated than what is needed in this two-dimensional study. In three-dimensional layered problems, each matrix component in a moment method solution consists of a double inverse Fourier transform (found using a plane wave spectral formulation) [3]. Only under very special conditions (see Appendix I of [4]), not possible for most practical testing and expansion functions, can this double integral be reduced to a one-dimensional Sommerfeld type integral. Hence, most previous asymptotic treatments of spectral integrals have dealt with the efficient computation of the Green's function [5], [6], which for a three (or two) dimensional layered problem can be written as a single semi-infinite integral of the Sommerfeld type [4]. In these approaches, after efficiently determining the space domain Green's function, one is still required to evaluate space domain integrals involving expansion and testing functions. The key difference between the asymptotic procedure developed here and those used previously for three-dimensional problems is that for distant expansion and testing functions, the entire matrix component is efficiently evaluated asymptotically in the spectral domain, without the need for subsequent space domain integration.

In Section II, the spectral domain technique is briefly reviewed and then, by considering the physical nature of modes on printed interconnects, an efficient asymptotic technique is developed for studying high-frequency effects in multiconductor printed interconnects. The asymptotic technique is then used to study leaky waves on two example structures, and the results are compared with data in the literature.

## II. Analysis

### A. Formulation

The spectral domain technique with a Galerkin moment method solution [1], [2] will be used to calculate the complex propagation constants for multiconductor printed transmission lines, such as that depicted in Fig. 1. This analysis results in a matrix equation with